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FUZZY UNBALANCED TRANSPORTATION PROBLEM BY USING MONTE CARLO METHOD

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ABSTRACT

In this article we consider fuzzy unbalanced transportation problem by using triangular fuzzy number. We find the initial solution by using fuzzy matrix minima method. The main objective of this paper is to find optimal solution to given unbalanced transportation problem using Monte Carlo Method i.e. by using triangular fuzzy random number. In general, transportation problems are solved with the assumptions that unit cost of transportation from each source to each destination, supply of the product at each source and demand at each destination are specified in a exact way i.e., in crisp environment. But in practice, many times we face the problem of incompleteness uncertain data, this is due to lack of knowledge about the considered system or changing nature of the world, the parameters of the transportation problem are not always exactly known and stable. Therefore we used the fuzzy logic to solve transportation problem. Fuzzy logic & techniques has been widely used in many areas such as engineering, business, mathematics, psychology, management, medicine and image processing and pattern recognition.

Keyword:Unbalance, Random Number, Monte Carlo Method.

INTRODUCTION

Transportation problem which has been used to solve different type of real life problems and generally studied in operation research field. Fuzzy logic deals with degrees of truth rather than the usual true or false (1 or 0) on which the modern computer technology is based. Fuzzy Set Theory gives the formalization of approximate reasoning, and preserves the original information contents of imprecision. Hitchcock (L 1941) first time developed the basic transportation problem. Appa(M 1973) discussed different method of the transportation problem. Prof.Zadeh (Z. L. A 1965)father of fuzzy mathematics introduced the concept of fuzzy numbers.Saad& Abbas (S. O. A 2003)discussed an algorithm for solving the transportation problems in fuzzyenvironment. Das &Baruah(K 2007) proposed Vogel's approximation method to find the fuzzyinitial basic feasible solution of fuzzy transportation problems in which all the parameters arerepresented by triangular fuzzy numbers. Basirzadeh (H 2011) used the classical algorithms tofind the fuzzy optimal solution of fully fuzzy transportation problems by transforming the fuzzyparameters into crisp parameters. Kaur& Kumar (K. A. A 2011)proposeda new method for the fuzzy transportation problems using ranking function.Deepika Rani, T R Gulati&Amit Kumar (Deepika Rani 2014) developed method for unbalanced transportation problems in fuzzy environment. Ali Ebrahimnejad (Ebrahimnejad 2014)used the values of transportation costs are represented by generalized trapezoidal fuzzy numbers and the values of supply and demand of products are represented by real numbers. Here we concluded that once the ranking function is chosen, the FTPis converted into crisp one, which is easily solved by the standardtransportation algorithms.

This paper is organized as follows. In section 2, the triangular membership function isdefined. In the next section, the general transportation problem with fuzzy triangularnumbers is discussed. This is followed by the solution of transportation problem using fuzzy triangular numbers in section 4. Section 5 illustrates the solution of transportation problem through a numerical example and Matlabprogramme is given. Finally, in section 7 conclusions are given.

Preliminaries:

Fuzzy set: A fuzzy set is defined by $\{(x, \mu_A(x)): x \in A, \mu_A(x) \in [0,1]\}$. In the pair $(x,\mu_A(x))$,the first element x belong to the classical set A, the second element $\mu_A(x)$, belong to the interval[0, 1], called Membership function.

Normality:A fuzzy set is called **normal** if its core is nonempty. In other words, there is at least one point x∈*X*with $\mu_A(x) = 1$.

Fuzzy Number: A fuzzy set on R must possess at least the following three properties to qualifyas a fuzzy number,

(i) A must be a normal fuzzy set;

(ii) α must be closed interval for every α∈ [0, 1]

(iii) The support of α, must be bounded

Triangular Fuzzy Number:A triangular fuzzy number A or simply triangular number represented with three points as follows(a1, aM, a2) holds the following conditions

$(i)\ a1\ to\ aM\ is\ increasing\ function$

$(ii)\ \ aM\ to\ a2\ is\ decreasing\ function$

$(iii)\ a1\ \le aM \le a2$.

This representation is interpreted as membership functions

$$\mu_A(x) = \begin{cases} \frac{x-a1}{aM-a1} & a1 \le x \le aM \\ \frac{x-a2}{aM-a2} & aM \le x \le a2 \\ 0 & otherwise \end{cases}$$

where [a1; a2] is the supporting interval and the point (aM; 1) is the peak.

Tabular Representation: Suppose there are *m* factories and *n* warehouses then transportation problem is usually represented in tabular form

Table 1 Tabular Representation of Crisp Transportation Problem

Destination / Origin	D1	D2	D3	-------	Dn	Supply
O1	C_{11}	C_{12}	C_{13}	-------	C_{1n}	A1
O2	C_{21}	C_{22}	C_{23}	-------	C_{2n}	A2
O3	C_{31}	C_{32}	C_{33}	-------	C_{3n}	A3
⋮	⋮	⋮	⋮		⋮	⋮
Om	C_{m1}	C_{m2}	C_{m3}	-------	C_{mn}	Am
Demand	B1	B2	B3	-------	Bn	$\sum_{i=1}^{n} Bi = \sum_{j=1}^{m} Aj$

Basic feasible solution: A feasible solution to a*m*-origin and *n*- destination problem is said to be basic if the number of positive allocation are *m* + *n* -1.

Theorem: There always exists an optimal solution to a balanced transportation problem.

MATRIX MINIMA METHOD

The initial basic feasible solution obtained by this method usually gives a lower beginning cost.

Step 1: Start with the lowest cost entry in the cell and allocate as much as possible.

Step 2: Move to the next lowest cost cell and make an allocation in the view of the remaining capacity and requirement of its row and column. In case there is tie for lowest cost cell during any allocation we can exercise our judgment and we arbitrarily choose cell for allocation.

Step 3: Above procedure repeated till all row and column requirements are satisfied.

UNBALANCED TRANSPORTATION PROBLEM

A transportation problem is said to unbalanced if total supply at sources is not equal to the total demand of the destinations.

There are two cases in unbalanced T.P.

Case (1): Supply exceeds demand:

In this case the total capacity of sources is greater than the total requirement of the destination i.e. in this case $\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j$

To make such problems balanced we add dummy row or dummy source in transportation table with zero transportation cost.

Case (2):Demand exceeds supply:

In this case the total demand of destination is greater than the total capacity of the destination i.e. in this case $\sum_{i=1}^{m} a_i < \sum_{j=1}^{n} b_j$

To make such problems balanced we add dummy column in transportation table with zero transportation cost.

TRANSPORTATION PROBLEM INTO CRISP LINEAR PROGRAMMING PROBLEM

Let there be *m* origins, i^{th} origin possessing A_i units (see table 1) of a certain product, whereas there are *n* destinations with destination on *j* requiring B_j units. Let C_{ij} be the cost of shipping one unit product from i^{th} origin to j^{th} destination and 'X_{ij}' be the amount to be shipped from i^{th} origin to j^{th} destination. Here we assume that $\sum A_i \ge \sum B_j$ *i= 1,2,..,m* and *j*= 1,2,...,*n*.

LPP formulation of above transportation problem is given below

$Min\ Z = \sum_{i=1}^{m} \sum_{j=1}^{n} X_{ij} C_{ij}$ (Objective function)

Subject to

$\sum_{j=1}^{n} x_{ij} \leq A_i$ for $i = 1,2,.....,m$

$\sum_{i=1}^{m} x_{ij} \geq B_j$ for $j = 1,2,.....,n$

The problem is to determine non negative values of X_{ij} satisfying both availability constraints.

TRANSPORTATION PROBLEM INTO FUZZY LINEAR PROGRAMMING PROBLEM

Let there be m origins, i^{th} origin possessing $\overline{A_i}$ units (see table 2) of a certain product, whereas there are n destinations with destination on j requiring $\overline{B_j}$ (see table 2) units. Let $\overline{C_{ij}}$ be the cost of shipping one unit product from ith origin to j^{th} destination and '$\overline{X_{ij}}$' be the amount to be shipped from i^{th} origin to j^{th} destination. Here we assume that $\sum \overline{A_i} \geq \sum \overline{B_j}$ $i= 1,2,..,m$ and $j= 1,2,...,n$. (see table 2)

LPP formulation of above transportation problem is given below

Min $z = \sum_{i=1}^{m} \sum_{j=1}^{n} \overline{X_{ij} C_{ij}}$ (objective function)

Subject to

$\sum_{j=1}^{n} \overline{X_{ij}} \leq \overline{A_i}$ for $i= 1, 2,....., m$

$\sum_{i=1}^{m} \overline{X_{ij}} \geq \overline{B_j}$ for $j = 1, 2,....., n$

where $\overline{A_i}$, $\overline{B_j}$, $\overline{C_{ij}}$, $\overline{X_{ij}}$ are all fuzzy triangular number.

Table 2 Tabular Representation of fuzzy Transportation Problem By Using triangular Fuzzy number

Destination / Origin	D1	D2	D3	-------	Dn	Supply
O1	$[C_{11}-d, C_{11}, C_{11}+d]$	$[C_{12}-d, C_{12}, C_{12}+d]$	$[C_{13}-d, C_{13}, C_{13}+d]$	-------	$[C_{1n}-d, C_{1n}, C_{1n}+d]$	$[a_1-d, a_1, a_1+d]$
O2	$[C_{21}-d, C_{21}, C_{21}+d]$	$[C_{22}-d, C_{22}, C_{22}+d]$	$[C_{23}-d, C_{23}, C_{123}+d]$	-------	$[C_{2n}-d, C_{2n}, C_{2n}+d]$	$[a_2-d, a_2, a_2+d]$
O3	$[C_{31}-d, C_{31}, C_{31}+d]$	$[C_{32}-d, C_{32}, C_{32}+d]$	$[C_{33}-d, C_{33}, C_{33}+d]$	-------	$[C_{3n}-d, C_{3n}, C_{3n}+d]$	$[a_3-d, a_3, a_3+d]$
¦	¦	¦	¦	¦	¦	¦
Om	$[C_{m1}-d, C_{m1}, C_{m1}+d]$	$[C_{m2}-d, C_{m2}, C_{m2}+d]$	$[C_{m3}-d, C_{m3}, C_{m3}+d]$	-------	$[C_{mn}-d, C_{mn}, C_{mn}+d]$	$[a_m-d, a_m, a_m+d]$
Demand	$[b_1-d, b_1, b_1+d]$	$[b_2-d, b_2, b_2+d]$	$[b_3-d, b_3, b_3+d]$	-------	$[bn-d, bn, bn+d]$	$\sum_{i=1}^{n}[bi-d, bi, bi+d] - \sum_{j=1}^{m}[ai-d, ai, ai-d] = [-k, 0, k]$

MONTE CARLO METHOD

Monte Carlo Method gives approximate solution to fuzzy optimization problem. It is a numerical method that makes use of random number to solve mathematical problem for which an analytical solution is not known; that is trough random number experiment on computer. To compare two random triangular fuzzy number say $\bar{X} = (x_1/x_2/x_3)$ and $\bar{Y} = (y_1/y_2/y_3)$ we find here α cut say Xα and Yα. If each α cut Xα is less than or equal to each α-cut of Yα (α>0.5) then we can say that fuzzy number $\bar{X} \leq \bar{Y}$.

RANDOM NUMBER

Monte Carlo Method is deals with use of random number. We use Matlab function r = rand() to generate random number in the interval [0,1], then by using function (b-a)*r + a we can generate random number in any interval [a,b]. By using sort and reshape function of Matlab we can convert these random numbers into fuzzy triangular numbers.

INTERVAL CONTAINING SOLUTION

Range of interval is very important because exact selection of this interval will make Monte Carlo Method more efficient. If interval is too large then too many of random number rejected and if it is very small then we can miss optimum solution. Suppose there are m equation in $n(x_1,x_2,....x_n)$ variable then put n-1 variable equal to zero find value of x_1 similarly find the values of $x_2,x_3,...,x_n$ by equating ni1 variable equal to zero. Finally to obtain upper bound take maximum of x_1, x_2 ...x_n.

Numerical Example:

Consider the following unbalanced transportation problem having four destinations and three origins.

Table 3 Crisp Unbalance Transportation Problem

Destination / Origin	D1	D2	Supply
O1	40 (2)	70	2
O2	60 **(1)**	30 (1)	3
Demand	3	1	4 \ 5

We convert the above unbalance transportation problem into balance by adding one column

Destination / Origin	D1	D2	D3	Supply
O1	40 (2)	70	0	2
O2	60 **(1)**	30 **(1)**	0 **(1)**	3
Demand	3	1	1	5

Minimum Transportation cost is 170

Destination / Origin	D1	D2	D3	Supply
O1	[39,40,41] **(1,2,3)**	[69,70,71]	[-1,0,1]	[1,2,3]
O2	[59,60,61] **(0.5,1,1.5)**	[29,30,31] **(0.5,1,1.5)**	[-1,0,1] **(0.5,1,1.5)**	[2,3,4]
Demand	[2,3,1]	[0.5,1,1.5]	[0.5,1,1.5]	

Table 5 Fuzzy Transportation Problem

[39,80,123]+[29,60,91.5]+[14,30,46]+[-1.5,0,1.5]=[80.5,170,262]
Minimum Transportation Costis ***170.83*** *approximately.*

FUZZY MONTE CARLO METHOD

Step I - Fuzzy Linear Programming Problem:

Min z = $(39/40/41)\overline{x1}$ + $(69/70/71)\overline{x2}$+$(59/60/61)\overline{x4}$ +$(29/30/31)\overline{x5}$

Subject to

$\overline{x1} + \overline{x2} \leq (1/2/3)$

$\overline{x3} + \overline{x4} \leq (2/3/4)$

$\overline{x1} + \overline{x3} \geq (2/3/4)$

$\overline{x2} + \overline{x4} \geq (0.5/1/1.5)$

Where $\overline{x1}, \overline{x2}, \overline{x3}, \overline{x4} \geq 0$

MatlabProgramme:

```
clc
r1=rand(9999,1);
'Enter interval a & b'
a=input('');
b=input('');
x=(b-a)*r3+a;
for i= 1:3333
for j= 1:3333
for p= 1:3333
for q= 1:3333
X1=[x(i,1) x(i,2) x(i,3)]+[x(j,1) x(j,2) x(j,3)];
X2=[x(i,1) x(i,2) x(i,3)]+[x(p,1) x(p,2) x(p,3)];
X3=[x(p,1) x(p,2) x(p,3)]+[x(q,1) x(q,2) x(q,3)];
X4=[x(q,1) x(q,2) x(q,3)]+[x(j,1) x(j,2) x(j,3)];
a3=min([a1*x(i,1),a1*x(i,2),a1*b1*x(i,1),b1*x(i,2),b1*x(i,3),c1*x(i,1),c1*x(i,2),c1*x(i,3)]);
b3=b1*x(i,2);
c3=max([a1*x(i,1),a1*x(i,2),a1*x(i,3),b1*x(i,1),b1*b1*x(i,3),c1*x(i,1),c1*x(i,2),c1*x(i,3)]);
s1=[a3  b3  c3];
   a1=69;
   b1=70;
   c1=71;
a5=min([a1*x(p,1),a1*x(p,2),a1*x(p,3),b1*b1*x(p,3),c1*x(p,1),c1*x(p,2),c1*x(p,3)]);
b5=b1*x(p,2);
c5=max([a1*x(p,1),a1*x(p,2),a1*x(p,3),b1*x(p,1),b1*x(p,2),b1*x(p,3),c1*x(p,1),c1*x(p,2),c1*x(p,3)]);
s3=[a5  b5  c5];
   a1=29;
   b1=30;
   c1=31;
a6=min([a1*x(q,1),a1*x(q,2),a1*x(q,3),b1*x(q,1),b1*x(q,2),b1*x(q,3),c1*x(q,1),c1*x(q,2),c1*x(q,3)]);
b6=b1*x(q,2);
s4=[a6  b6  c6];
s=s1+s2+s3+s4
X11=[x(i,1) x(i,2) x(i,3)]
X12=[x(j,1) x(j,2) x(j,3)]
X13=[x(p,1) x(p,2) x(p,3)]
X14=[x(q,1) x(q,2) x(q,3)]
end
```

Solution:

Minimum Transportation Cost :(**41.9392 170.9264 290.6067)**

X11 =(0.2589 0.9843 1.4900)
X12 =(0.2436 0.8853 1.4612)
X13 =(0.1275 0.6675 1.3045)
X14 =(0.2589 0.9843 1.4900)

CONCLUSION AND FUTURE WORK

In this articlewe discussed a method of fuzzy unbalancedtransportation problem by using Monte Carlo Method i.e by takingrandom triangular fuzzy number. In this method, through a numerical example we can conclude that Monte Carlo method gives best approximate solution to given fuzzy transportation problem. In future; we want to extend our work doing more research by using trapezoidal fuzzy number.

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